次の不定積分を求めよ。

(1) $\displaystyle\int \frac{x}{(x+1)^2}\,dx$

(2) $\displaystyle\int x\sqrt{x-1}\,dx$

(3) $\displaystyle\int sin\,x\,cos^2x\,dx$

(4) $\displaystyle\int 2x(x^2+1)^4\,dx$

(5) $\displaystyle\int \frac{2x-1}{x^2-x+5}\,dx$

(6) $\displaystyle\int \frac{log\,x}{x}\,dx$

以下、積分定数を C とする。

(1)　$t = x+1$ とおくと $dt = dx$

$$\int \frac{x}{(x+1)^2}\,dx$$
$$= \int \frac{t-1}{t^2}\,dt$$
$$= \int \frac{1}{t} - \frac{1}{t^2}\,dt$$
$$= log|t| - \frac{1}{t} + C = log|x+1| - \frac{1}{x+1} + C$$

(2)　$t = \sqrt{x-1}$ とおくと

$$t^2 = x-1$$
$$2t \cdot dt = dx$$
$$\int x\sqrt{x-1}\,dx$$
$$= \int (t^2+1)t \cdot 2t \cdot dt$$
$$= 2\int t^4 + t^2\,dt$$
$$= \frac{2}{5}t^5 + \frac{2}{3}t^3 + C$$
$$= \frac{2}{5}(x-1)^2\sqrt{x-1} + \frac{2}{3}(x-1)\sqrt{x-1} + C$$

(3)　$cos\,x = t$ とおくと、$-sin\,x\,dx = dt$

$$\int sin\,x\,cos^2x\,dx$$
$$= \int -t^2\,dt$$
$$= -\frac{1}{3}t^3 + C = -\frac{1}{3}cos^3x + C$$

(4)　$x^2+1 = t$ とおくと、$2x\,dx = dt$

$$\int 2x(x^2+1)^4\,dx$$
$$= \int t^4\,dt$$
$$= \frac{1}{5}t^5 + C = \frac{1}{5}(x^2+1) + C$$

(5)　$x^2 - x + 5 = t$ とおくと、

$$(2x-1)dx = dt$$

$$\int \frac{2x-1}{x^2-x+5}\,dx$$
$$= \int \frac{1}{t}\,dt$$
$$= log|t| + C = log|x^2-x+5| + C$$

(6)　$log\,x = t$ とおくと、

$$\frac{1}{x}dx = dt$$

$$\int \frac{log\,x}{x}\,dx$$
$$= \int t\,dt$$
$$= \frac{1}{2}t^2 + C = \frac{1}{2}(log\,x)^2 + C$$

次の不定積分を求めよ。

(1) $\int (x+1)(x^2+2x-1)^3\,dx$

(2) $\int \frac{2(x-2)}{\sqrt{x^2-4x}}\,dx$

(3) $\int tan\,x\,dx$

(4) $\int (e^x+1)(e^x+x)\,dx$

(5) $\int sin^3 x\,dx$

(6) $\int \frac{1}{x\,log\,x}\,dx$

不定積分の計算5（置換積分） No2 解答

以下、積分定数を C とする。

(1) $x^2+2x-1=t$ とおくと

$$(2x+2)dx=dt$$
$$(x+1)dx=\frac{1}{2}dt$$
$$\int (x+1)(x^2+2x-1)^3\,dx$$
$$=\int \frac{1}{2}t^3\,dt$$
$$=\frac{1}{8}t^4+C=\frac{1}{8}(x^2+2x-1)^4+C$$

(2) $\sqrt{x^2-4x}=t$ とおくと

$$x^2-4x=t^2$$
$$(2x-4)dx=2t\,dt$$
$$\int \frac{2(x-2)}{\sqrt{x^2-4x}}\,dx$$
$$=\int 2\,dt$$
$$=2t+C=2\sqrt{x^2-4x}+C$$

(3) $\cos x=t$ とおくと、$-\sin x\,dx=dt$

$$\int \tan x\,dx$$
$$=\int \frac{\sin x}{\cos x}\,dx$$
$$=\int \frac{1}{t}\,dt$$
$$=-\log|t|+C=-\log|\cos x|+C$$

(4) $e^x+x=t$ とおくと、$(e^x+1)dx=dt$

$$\int (e^x+1)(e^x+x)\,dx$$
$$=\int t\,dt$$
$$=\frac{1}{2}t^2+C=\frac{1}{2}(e^x+x)+C$$

(5) $\cos x=t$ とおくと、$-\sin x\,dx=dt$

$$\int \sin^3 x\,dx$$
$$=\int \sin x\cdot\sin^2 x\,dx$$
$$=\int \sin x\cdot(1-\cos^2 x)\,dx$$
$$=\int t^2-1\,dt$$
$$=\frac{1}{3}t^3-t+C=\frac{1}{3}\cos^3 x-\cos x+C$$

(6) $\log x=t$ とおくと、

$$\frac{1}{x}dx=dt$$
$$\int \frac{1}{x\log x}\,dx$$
$$=\int \frac{1}{t}\,dt$$
$$=\log|t|+C=\log|\log x|+C$$

不定積分の計算5（置換積分） No3

次の不定積分を求めよ。

(1) $\int 3(x+1)(x-1)(x^3-3x)^3\,dx$

(2) $\int sin\,x\sqrt{cos\,x}\,dx$

(3) $\int \frac{1}{tan\,x}\,dx$

(4) $\int \frac{e^x}{e^x+1}\,dx$

(5) $\int \frac{log(log\,x)}{x}\,dx$

(6) $\int 2^x(2^x+1)^2\,dx$

不定積分の計算5（置換積分） No3 解答

以下、積分定数を C とする。

(1) $x^3-3x=t$ とおくと、$(3x^2-3)dx=dt$

$$\int 3(x+1)(x-1)(x^3-3x)^3\,dx$$
$$=\int t^3\,dt$$
$$=\frac{1}{4}t^4+C=\frac{1}{4}(x^3-3x)^4+C$$

(2) $\sqrt{\cos x}=t$ とおくと、

$$\cos x=t^2$$
$$-\sin x=2t\cdot dt$$
$$\int \sin x\sqrt{\cos x}\,dx$$
$$=\int -2t^2\,dt$$
$$=-\frac{2}{3}t^3+C=-\frac{2}{3}\cos x\sqrt{\cos x}+C$$

(3) $\sin x=t$ とおくと、$\cos x\,dx=dt$

$$\int \frac{1}{\tan x}\,dx$$
$$=\int \frac{\cos x}{\sin x}\,dx$$
$$=\int \frac{1}{t}\,dx$$
$$=\log|t|+C=\log|\sin x|+C$$

(4) $e^x+1=t$ とおくと、$e^x dx=dt$

$$\int \frac{e^x}{e^x+1}\,dx$$
$$=\int \frac{1}{t}\,dt$$
$$=\log|t|+C=\log(e^x+1)+C$$

(5) $\log x=t$ とおくと、$\frac{1}{x}\,dx=dt$

$$\int \frac{\log(\log x)}{x}\,dx$$
$$=\int \log t\,dt$$
$$=t\log t-t+C$$
$$=\log x\log(\log x)-\log x+C$$

(6) $2^x+1=t$ とおくと、$\frac{2^x}{\log 2}\,dx=dt$

$$\int 2^x(2^x+1)^2\,dx$$
$$=\int \frac{t^2}{\log 2}\,dt$$
$$=\frac{t^3}{3\log 2}+C=\frac{(2^x+1)^3}{3\log 2}+C$$