

展開の工夫

1. 次の式を展開せよ。

(1) $(x^2 - 2x + 5)(x^2 - 2x - 2)$

(2) $(x^2 + 3x - 2)(x^2 - 2x - 2)$

(3) $(x + y - z)(x - y + z)$

(4) $(a + b + c)^2$

2. 次の式を展開せよ。

(1) $(x + 3)(x^2 + 9)(x - 3)$

(2) $(a + 2b)^2(a - 2b)^2$

(3) $(x + 1)(x + 2)(x - 3)(x - 4)$

3. 次の式を展開せよ。

(1) $(x + 2)(x - 3)(x^2 - 2x + 4)(x^2 + 3x + 9)$

(2) $(x - 2)^3(x + 2)^3$

(3) $(x - 1)(x^2 + 1)(x + 1)(x^4 + 1)$

(4) $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

4. 次の式を展開せよ。

(1) $(x^2 + 2x - 3)(x^2 + 2x + 3)$

(2) $(x^2 + 2x - 1)(x^2 - 2x - 1)$

(3) $(x + y + z)(x - y - z)$

5. 次の式を展開せよ。

(1) $(x - 1)(x^2 + 1)(x + 1)$

(2) $(x + 3)^2(x - 3)^2$

(3) $(x + 2)(x + 3)(x - 6)(x - 5)$

1. 次の式を展開せよ。

$$\begin{aligned}(1) & (x^2 - 2x + 5)(x^2 - 2x - 2) \\ &= \{(x^2 - 2x) + 5\} \{(x^2 - 2x) - 2\} \\ &= (x^2 - 2x)^2 + 3(x^2 - 2x) - 10 \\ &= x^4 - 4x^3 + 4x^2 + 3x^2 - 6x - 10 \\ &= x^4 - 4x^3 + 7x^2 - 6x - 10\end{aligned}$$

$$\begin{aligned}(2) & (x^2 + 3x - 2)(x^2 - 2x - 2) \\ &= (x^2 - 2 + 3x)(x^2 - 2 - 2x) \\ &= (x^2 - 2)^2 + x(x^2 - 2) - 6x \\ &= x^4 - 4x^2 + 4 + x^3 - 2x - 6x \\ &= x^4 + x^3 - 4x^2 - 8x + 4\end{aligned}$$

$$\begin{aligned}(3) & (x + y - z)(x - y + z) \\ &= \{x + (y - z)\}\{x - (y - z)\} \\ &= x^2 - (y - z)^2 \\ &= x^2 - y^2 - z^2 + 2yz \\ &(x^2 - y^2 + 2yz - z^2 \text{ でもOK})\end{aligned}$$

$$\begin{aligned}(4) & (a + b + c)^2 \\ &= \{a + (b + c)\}^2 \\ &= a^2 + 2(b + c)a + (b + c)^2 \\ &= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca\end{aligned}$$

2. 次の式を展開せよ。

$$\begin{aligned}(1) & (x + 3)(x^2 + 9)(x - 3) \\ &= (x + 3)(x - 3)(x^2 + 9) \\ &= (x^2 - 9)(x^2 + 9) \\ &= x^4 - 81\end{aligned}$$

$$\begin{aligned}(2) & (a + 2b)^2(a - 2b)^2 \\ &= (a + 2b)(a - 2b)(a + 2b)(a - 2b) \\ &= (a^2 - 4b^2)^2 \\ &= a^4 - 8a^2b^2 + 16b^4\end{aligned}$$

$$\begin{aligned}(3) & (x + 1)(x + 2)(x - 3)(x - 4) \\ &= (x + 1)(x - 3)(x + 2)(x - 4) \\ &= (x^2 - 2x - 3)(x^2 - 2x - 8) \\ &= (x^2 - 2x)^2 - 11(x^2 - 2x) + 24 \\ &= x^4 - 4x^3 + 4x^2 - 11x^2 + 22x + 24 \\ &= x^4 - 4x^3 - 7x^2 + 22x + 24\end{aligned}$$

3. 次の式を展開せよ。

$$\begin{aligned}(1) \quad & (x+2)(x-3)(x^2-2x+4)(x^2+3x+9) \\ &= (x+2)(x^2-2x+4)(x-3)(x^2+3x+9) \\ &= (x^3+8)(x^3-27) \\ &= x^6-19x^3-216\end{aligned}$$

$$\begin{aligned}(2) \quad & (x-2)^3(x+2)^3 \\ &= \{(x-2)(x+2)\}^3 \\ &= (x^2-4)^3 \\ &= (x^2)^3+3\cdot(x^2)^2\cdot(-4) \\ &\quad +3\cdot x^2\cdot(-4)^2+(-4)^3 \\ &= x^6-12x^4+48x^2-64\end{aligned}$$

$$\begin{aligned}(3) \quad & (x-1)(x^2+1)(x+1)(x^4+1) \\ &= (x-1)(x+1)(x^2+1)(x^4+1) \\ &= (x^2-1)(x^2+1)(x^4+1) \\ &= (x^4-1)(x^4+1) \\ &= x^8-1\end{aligned}$$

$$\begin{aligned}(4) \quad & (a+b+c)(a^2+b^2+c^2-ab-bc-ca) \\ &= \{a+(b+c)\}\{a^2-(b+c)a+b^2-bc+c^2\} \\ &= a^3+\{(b+c)-(b+c)\}a^2+\{b^2-bc+c^2-(b+c)^2\}a+(b+c)(b^2-bc+c^2) \\ &= a^3+(b^2-bc+c^2-b^2-2bc-c^2)a+b^3+c^3 \\ &= a^3+b^3+c^3-3bc\end{aligned}$$

4. 次の式を展開せよ。

$$\begin{aligned}(1) \quad & (x^2+2x-3)(x^2+2x+3) \\ &= (x^2+2x)^2-9 \\ &= x^4+4x^3+4x^2-9\end{aligned}$$

$$\begin{aligned}(2) \quad & (x^2+2x-1)(x^2-2x-1) \\ &= (x^2-1+2x)(x^2-1-2x) \\ &= (x^2-1)^2-4x^2 \\ &= x^4-2x^2+1-4x^2 \\ &= x^4-6x^2+1\end{aligned}$$

$$\begin{aligned}(3) \quad & (x+y+z)(x-y-z) \\ &= \{x+(y+z)\}\{x-(y+z)\} \\ &= x^2-(y+z)^2 \\ &= x^2-y^2-z^2+2yz\end{aligned}$$

5. 次の式を展開せよ。

$$\begin{aligned}(1) \quad & (x-1)(x^2+1)(x+1) \\ &= (x-1)(x+1)(x^2+1) \\ &= (x^2-1)(x^2+1) \\ &= x^4-1\end{aligned}$$

$$\begin{aligned}(2) \quad & (x+3)^2(x-3)^2 \\ &= \{(x+3)(x-3)\}^2 \\ &= (x^2-9)^2 \\ &= x^4-18x^2+81\end{aligned}$$

$$\begin{aligned}(3) \quad & (x+2)(x+3)(x-6)(x-5) \\ &= (x+2)(x-5)(x+3)(x-6) \\ &= (x^2-3x-10)(x^2-3x-18) \\ &= (x^2-3x)^2-28(x^2-3x)+180 \\ &= x^4-6x^3+9x^2-28x^2+84x+180 \\ &= x^4-6x^3-19x^2+84x+180\end{aligned}$$