

展開 No.1

次の式を展開しなさい。

(1) $(x + 2y)^3$

(3) $(2a - 3b)(4a^2 + 6ab + 9b^2)$

(5) $(a + b - c)^2$

(7) $(p^2 + p - 1)(p^2 - p - 1)$

(9) $(x - 1)(x - 2)(x - 3)(x - 4)$

(11) $(x^2 - 3x + 1)(x^2 + 3x - 1)$

(2) $(a + 1)(a - 2)(a^2 - a + 1)(a^2 + 2a + 4)$

(4) $(q^2 - q + 2)(q^2 + q - 2)$

(6) $(a + 2b)^2(a - 2b)^2$

(8) $(x + 2)(x^2 - 2x + 4)$

(10) $(a + 3)^3$

(12) $(x + y)(x^2 + y^2)(x - y)$

展開 No.1 (解答)

$$\begin{aligned}(1) \quad & (x+2y)^3 \\ &= x^3 + 3 \cdot x^2 \cdot 2y + 3 \cdot x \cdot (2y)^2 + (2y)^3 \\ &= x^3 + 6x^2y + 12xy^2 + 8y^3\end{aligned}$$

$$(3) \quad (2a-3b)(4a^2+6ab+9b^2) = 8a^3 - 27b^3$$

$$\begin{aligned}(5) \quad & (a+b-c)^2 \\ &= (a+b)^2 - 2(a+b)c + c^2 \\ &= a^2 + 2ab + b^2 - 2ac - 2bc + c^2 \\ &= a^2 + b^2 + c^2 + 2ab - 2bc - 2ca\end{aligned}$$

$$\begin{aligned}(7) \quad & (p^2+p-1)(p^2-p-1) \\ &= (p^2-1+p)(p^2-1-p) \\ &= (p^2-1)^2 - p^2 \\ &= p^4 - 2p^2 + 1 - p^2 \\ &= p^4 - 3p^2 + 1\end{aligned}$$

$$\begin{aligned}(9) \quad & (x-1)(x-2)(x-3)(x-4) \\ &= (x-1)(x-4)(x-2)(x-3) \\ &= (x^2-5x+4)(x^2-5x+6) \\ &= (x^2-5x)^2 + 6(x^2-5x) + 4(x^2-5x) + 24 \\ &= x^4 - 10x^3 + 25x^2 + 10x^2 - 50x + 24 \\ &= x^4 - 10x^3 + 35x^2 - 50x + 24\end{aligned}$$

$$\begin{aligned}(11) \quad & (x^2-3x+1)(x^2+3x-1) \\ &= \{x^2-(3x-1)\}(x^2+3x-1) \\ &= x^4 - (3x-1)^2 \\ &= x^4 - (9x^2 - 6x + 1) \\ &= x^4 - 9x^2 + 6x - 1\end{aligned}$$

$$\begin{aligned}(2) \quad & (a+1)(a-2)(a^2-a+1)(a^2+2a+4) \\ &= (a^3+1)(a^3-8) \\ &= a^6 - 7a^3 - 8\end{aligned}$$

$$\begin{aligned}(4) \quad & (q^2-q+2)(q^2+q-2) \\ &= \{q^2-(q-2)\}(q^2+q-2) \\ &= q^4 - (q-2)^2 \\ &= q^4 - (q^2 - 4q + 4) \\ &= q^4 - q^2 + 4q - 4\end{aligned}$$

$$\begin{aligned}(6) \quad & (a+2b)^2(a-2b)^2 \\ &= \{(a+2b)(a-2b)\}^2 \\ &= \{a^2 - 4b^2\}^2 \\ &= a^4 - 8a^2b^2 + 16b^4\end{aligned}$$

$$(8) \quad (x+2)(x^2-2x+4) = x^3 + 8$$

$$(10) \quad (a+3)^3 = a^3 + 9a^2 + 27a + 27$$

$$\begin{aligned}(12) \quad & (x+y)(x^2+y^2)(x-y) \\ &= (x+y)(x-y)(x^2+y^2) \\ &= (x^2-y^2)(x^2+y^2) \\ &= x^4 - y^4\end{aligned}$$

展開 No.2

次の式を展開しなさい。

(1) $(p-2)(p+2)(p+1)(p+5)$

(3) $(2x+y)(4x^2-2xy+y^2)$

(5) $(p+2)^3$

(7) $(a-b-c)(a+b-c)$

(9) $(x-2y)^3$

(11) $(x^2-1)(x^2+2)(x^2+1)(x^2-2)$

(2) $(x+y+z)(x+y-z)$

(4) $(x-2)(x+3)(x^2+2x+4)(x^2-3x+9)$

(6) $(a-3)(a^2+3a+9)$

(8) $(a^2+2a-3)(a^2-2a+3)$

(10) $(x+y+z)^2$

(12) $(3q+2r)^2(3q-2r)^2$

展開 No.2 (解答)

$$\begin{aligned}(1) & (p-2)(p+2)(p+1)(p+5) \\ &= (p-2)(p+5)(p+2)(p+1) \\ &= (p^2+3p-10)(p^2+3p+2) \\ &= (p^2+3p)^2+2(p^2+3p)-10(p^2+3p)-20 \\ &= p^4+6p^3+9p^2-8p^2-24p-20 \\ &= p^4+6p^3+p^2-24p-20\end{aligned}$$

$$(3) (2x+y)(4x^2-2xy+y^2) = 8x^3+y^3$$

$$\begin{aligned}(5) & (p+2)^3 \\ &= p^3+3\cdot p^2\cdot 2+3\cdot p\cdot 2^2+2^3 \\ &= p^3+6p^2+12p+8\end{aligned}$$

$$\begin{aligned}(7) & (a-b-c)(a+b-c) \\ &= (a-c-b)(a-c+b) \\ &= (a-c)^2-b^2 \\ &= a^2-b^2+c^2-2ac\end{aligned}$$

$$\begin{aligned}(9) & (x-2y)^3 \\ &= x^3-3\cdot x^2\cdot 2y+3\cdot x\cdot (2y)^2-(2y)^3 \\ &= x^3-6x^2y+12xy^2-8y^3\end{aligned}$$

$$\begin{aligned}(11) & (x^2-1)(x^2+2)(x^2+1)(x^2-2) \\ &= (x^2-1)(x^2+1)(x^2+2)(x^2-2) \\ &= (x^4-1)(x^4-4) \\ &= x^8-5x^4+4\end{aligned}$$

$$\begin{aligned}(2) & (x+y+z)(x+y-z) \\ &= (x+y)^2-z^2 \\ &= x^2+2xy+y^2-z^2\end{aligned}$$

$$\begin{aligned}(4) & (x-2)(x+3)(x^2+2x+4)(x^2-3x+9) \\ &= (x-2)(x^2+2x+4)(x+3)(x^2-3x+9) \\ &= (x^3-8)(x^3+27) \\ &= x^6+19x^3-216\end{aligned}$$

$$(6) (a-3)(a^2+3a+9) = a^3-27$$

$$\begin{aligned}(8) & (a^2+2a-3)(a^2-2a+3) \\ &= (a^2+2a-3)\{a^2-(2a-3)\} \\ &= a^4-(2a-3)^2 \\ &= a^4-(4a^2-12a+9) \\ &= a^4-4a^2+12a-9\end{aligned}$$

$$\begin{aligned}(10) & (x+y+z)^2 \\ &= (x+y)^2+2(x+y)z+z^2 \\ &= x^2+2xy+y^2+2xz+2yz+z^2 \\ &= x^2+y^2+z^2+2xy+2yz+2zx\end{aligned}$$

$$\begin{aligned}(12) & (3q+2r)^2(3q-2r)^2 \\ &= \{(3q+2r)(3q-2r)\}^2 \\ &= \{9q^2-4r^2\}^2 \\ &= 81q^4-72q^2r^2+16r^4\end{aligned}$$

展開 No.3

次の式を展開しなさい。

(1) $(x + 2y - z)^2$

(3) $(a - 2b)(a^2 + 2ab + 4b^2)$

(5) $(a + 4)(a^2 - 4a + 16)$

(7) $(p - 3)^3$

(9) $(x^2 + 3)(x^2 + 2)(x^2 - 3)(x^2 - 2)$

(11) $(x + y - z)(x - y + z)$

(2) $(a^2 + 2a - 3)(a^2 - 2a + 3)$

(4) $(2a + b + c)(2a - b - c)$

(6) $(a + 3b)^2(a - 3b)^2$

(8) $(x + 2)(x - 1)(x + 3)(x + 6)$

(10) $(x - 1)(x^2 - x + 1)(x + 1)(x^2 + x + 1)$

(12) $(2x + y)^3$

展開 No.3 (解答)

$$\begin{aligned}(1) & (x+2y-z)^2 \\ &= (x+2y)^2 - 2(x+2y)z + z^2 \\ &= x^2 + 4xy + 4y^2 - 2xz - 4yz + z^2 \\ &= x^2 + 4y^2 + z^2 + 4xy - 4yz - 2zx\end{aligned}$$

$$(3) (a-2b)(a^2+2ab+4b^2) = a^3 - 8b^3$$

$$(5) (a+4)(a^2-4a+16) = a^3 + 64$$

$$\begin{aligned}(7) & (p-3)^3 \\ &= p^3 + 3 \cdot p^2 \cdot (-3) + 3 \cdot p \cdot (-3)^2 - 3^3 \\ &= p^3 - 9p^2 + 27p - 27\end{aligned}$$

$$\begin{aligned}(9) & (x^2+3)(x^2+2)(x^2-3)(x^2-2) \\ &= (x^2+3)(x^2-3)(x^2+2)(x^2-2) \\ &= (x^4-9)(x^4-4) \\ &= x^8 - 13x^4 + 36\end{aligned}$$

$$\begin{aligned}(11) & (2x+y-z)(2x-y-z) \\ &= (2x-z+y)(2x-z-y) \\ &= (2x-z)^2 - y^2 \\ &= 4x^2 - y^2 + z^2 - 4xz\end{aligned}$$

$$\begin{aligned}(2) & (a^2+2a-3)(a^2-2a+3) \\ &= \{a^2+(2a-3)\}\{a^2-(2a-3)\} \\ &= a^4 - (2a-3)^2 \\ &= a^4 - 4a^2 + 12a - 9\end{aligned}$$

$$\begin{aligned}(4) & (2a+b+c)(2a-b-c) \\ &= \{2a+(b+c)\}\{2a-(b+c)\} \\ &= 4a^2 - (b+c)^2 \\ &= 4a^2 - b^2 - c^2 - 2bc\end{aligned}$$

$$\begin{aligned}(6) & (a+3b)^2(a-3b)^2 \\ &= \{(a+3b)(a-3b)\}^2 \\ &= (a^2-9b^2)^2 \\ &= a^4 - 18a^2b^2 + 81b^4\end{aligned}$$

$$\begin{aligned}(8) & (x+2)(x-1)(x+3)(x+6) \\ &= (x+2)(x+3)(x-1)(x+6) \\ &= (x^2+5x+6)(x^2+5x-6) \\ &= (x^2+5x)^2 - 36 \\ &= x^4 + 10x^3 + 25x^2 - 36\end{aligned}$$

$$\begin{aligned}(10) & (x-1)(x^2-x+1)(x+1)(x^2+x+1) \\ &= (x-1)(x^2+x+1)(x+1)(x^2-x+1) \\ &= (x^3-1)(x^3+1) \\ &= x^6 - 1\end{aligned}$$

$$\begin{aligned}(12) & (2x+y)^3 \\ &= (2x)^3 + 3 \cdot (2x)^2 \cdot y + 3 \cdot 2x \cdot y^2 + y^3 \\ &= 8x^3 + 12x^2y + 6xy^2 + y^3\end{aligned}$$

展開 No.4

次の式を展開しなさい。

(1) $(x-1)(x-3)(x-5)(x-7)$

(3) $(3x-y)(9x^2+3xy+y^2)$

(5) $(a+b)(a-b)^2(a-3b)$

(7) $(a^2-2a-3)(a^2+2a-3)$

(9) $(2a^2-1)(2a^2+3)(2a^2+1)(2a^2-3)$

(11) $(2a+3b)^3$

(2) $(a-b-c)^2$

(4) $(2x-1)^3$

(6) $(x-3)^2(x+3)^2$

(8) $(x^2-2x+4)(x-2)(x+2)(x^2+2x+4)$

(10) $(x^2+y^2-1)(x^2-y^2+1)$

(12) $(2x+3)(4x^2-6x+9)$

展開 No.4 (解答)

$$\begin{aligned}(1) & (x-1)(x-3)(x-5)(x-7) \\ &= (x-1)(x-7)(x-3)(x-5) \\ &= (x^2-8x+7)(x^2-8x+15) \\ &= (x^2-8x)^2+22(x^2-8x)+105 \\ &= x^4-16x^3+64x^2+22x^2-176x+105 \\ &= x^4-16x^3+86x^2-176x+105\end{aligned}$$

$$(3) (3x-y)(9x^2+3xy+y^2) = 27x^3 - y^3$$

$$\begin{aligned}(5) & (a+b)(a-b)^2(a-3b) \\ &= (a+b)(a-3b)(a-b)^2 \\ &= (a^2-2ab-3b^2)(a^2-2ab+b^2) \\ &= (a^2-2ab)^2-2b^2(a^2-2ab)-3b^4 \\ &= a^4-4a^3b+4a^2b^2-2a^2b^2+4ab^3-3b^4 \\ &= a^4-4a^3b+2a^2b^2+4ab^3-3b^4\end{aligned}$$

$$\begin{aligned}(7) & (a^2-2a-3)(a^2+2a-3) \\ &= (a^2-3-2a)(a^2-3+2a) \\ &= (a^2-3)^2-4a^2 \\ &= a^4-6a^2+9-4a^2 \\ &= a^4-10a^2+9\end{aligned}$$

$$\begin{aligned}(9) & (2a^2-1)(2a^2+3)(2a^2+1)(2a^2-3) \\ &= (2a^2-1)(2a^2+1)(2a^2+3)(2a^2-3) \\ &= (4a^4-1)(4a^4-9) \\ &= 16a^8-40a^4+9\end{aligned}$$

$$\begin{aligned}(11) & (2a+3b)^3 \\ &= (2a)^3+3\cdot(2a)^2\cdot 3b+3\cdot 2a\cdot(3b)^2+(3b)^3 \\ &= 8a^3+36a^2b+54ab^2+27b^3\end{aligned}$$

$$\begin{aligned}(2) & (a-b-c)^2 \\ &= (a-b)^2-2c(a-b)+c^2 \\ &= a^2-2ab+b^2-2ca+2bc+c^2 \\ &= a^2+b^2+c^2-2ab+2bc-2ca\end{aligned}$$

$$\begin{aligned}(4) & (2x-1)^3 \\ &= (2x)^2-3\cdot(2x)^2\cdot 1+3\cdot 2x\cdot 1-1^3 \\ &= 8x^2-12x^2+6x-1\end{aligned}$$

$$\begin{aligned}(6) & (x-3)^2(x+3)^2 \\ &= \{(x-3)(x+3)\}^2 \\ &= (x^2-9)^2 \\ &= x^4-18x^2+81\end{aligned}$$

$$\begin{aligned}(8) & (x^2-2x+4)(x-2)(x+2)(x^2+2x+4) \\ &= (x+2)(x^2-2x+4)(x-2)(x^2+2x+4) \\ &= (x^3+8)(x^3-8) \\ &= x^6-64\end{aligned}$$

$$\begin{aligned}(10) & (x^2+y^2-1)(x^2-y^2+1) \\ &= \{x^2+(y^2-1)\}\{x^2-(y^2-1)\} \\ &= x^4-(y^2-1)^2 \\ &= x^4-y^4+2y^2-1\end{aligned}$$

$$(12) (2x+3)(4x^2-6x+9) = 8x^3 + 27$$